**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:- B) 0.2676**

μ=45min, σ=8min

μ=10min after =45+10=55

Car will be ready in 1hr=60 min (x)

Z = x-μ/σ

= 60-55/8

= 0.625

1=stats. norm .cdf (z)

1=stats. norm. cdf (0.625)

=0.2659

P value for z score is 0.2659 approx to 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ= 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans** :- 68% of the data falls within one standard deviation of the mean (μ+σ)

Here, μ = 38

σ= 6

Z score =(X-μ)/σ

Z score for 44 =(44-38)/6=1

In python -stats. norm. cdf (1)=84.13=15.87%

People Above 44 age =100-84.13 = 15.87%

Given 400 clerical employes ,that is calculating 15.87% of 400 = 336.52 out of 400

=400-336.52

=63.49~63(approx)

Z score for 38 = (38-38)/6=0

In python. Stats.norm.cdf(1)=50%

Hence People between 38 & 44 age =84.13-50 = 34.13 % = 137 out of 400

**B**.A training program for employees under the age 30 at the center would be expected to attract about 36 employees is

**Ans**:- **True.**

Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15   %   ≈ 36 out of 400

(Z = (X - µ) / ϭ

P(X≤30) = P(Z ≤(30-38) / 6) = p (Z ≤ 1.33) = 0.0918 (Using Z-Table)

Expected count = 0.0918\*400 = 36.72)

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans** :-  *2X*1 is simply a large scale version of the random variable X , if *X*1 is normally distributed then *2X*1 is also normally distributed.

*X*1 and *X*2 are normal distributed, the associated sums and random samples are exactly normal, with appropriate parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans** :- since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The probability of getting value between a and b is: 0.99.

Therefore, probability of getting value outside a and b is: 1-0.99 = 0.01.

α = 1-0.99 / 2 = 0.005

Using Z-table, for probability 0.005 the Z value is: -2.57

Z = (X - µ) / ϭ

Rearranging formula to find the value of X:

X = (Z \* ϭ) + µ

For Z(-0.005) : Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

For Z(0.005 ) : Z(0.005)\*20+100 = (-2.57)\*20+100 = 48.6

**Option D is correct**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans** :-Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company.

**Ans** :- 5th percentile of profit (in Million Rupees) is 202

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans**:-

Probability of Division 1 making a loss P(X<0)

**Ans: 0.047**

Probability of Division 2 making a loss P(X<0)

**Ans: 0.040**